RELATIONSHIPS BETWEEN PROSPECTIVE MATHEMATICS TEACHERS' BELIEFS AND TPACK

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We examine relationships between prospective teachers' (PTs') beliefs about the nature of mathematics, learning and teaching mathematics, and the use of technology, and their knowledge of how to use technology to teach and learn mathematics. We interviewed 4 PTs and used Ernest's (1989) classification of beliefs and Goos, Galbraith, Renshaw, and Geiger's (2003) perspectives of technology to uncover PTs' beliefs. We examined PTs' knowledge of using technology by conducting a task-based interview based on the TPACK framework (Mishra & Koehler, 2006). We found there appeared to be relationships between PTs' beliefs about the nature of mathematics, learning and teaching mathematics, and the use of technology and their content knowledge, pedagogical content knowledge, and technological pedagogical content knowledge respectively.

Keywords: Teacher Beliefs; Teacher Education-Preservice; Teacher Knowledge; Technology

One of the guiding principles in the National Council of Teachers of Mathematics' (NCTM, 2014) *Principles to Actions: Ensuring Mathematics Success for All*is the use of tools and technology to explore and make sense of mathematics, reason mathematically, and communicate mathematical thinking. NCTM suggests when tools and technology are used appropriately they support effective teaching and promote meaningful learning. However, some teachers are reluctant to use technology to teach mathematics or do not use it in meaningful ways (Ertmer, 2005). Ertmer (1999) describes two types of barriers to teachers' integration of technology. First-order barriers are obstacles that can be eliminated if money is allocated. These barriers include resources such as access to digital tools, software, Internet, and time to plan and teach technology-based lessons. First-order barriers also include technology training and support, which contribute to teachers' knowledge of technology and how to integrate it into their practice. Second-order barriers are "typically rooted in teachers' underlying beliefs about teaching and learning and may not be immediately apparent to others or even to the teachers themselves" (Ertmer, 1999, p. 51). Second-order barriers are less tangible, more personal, and more deeply ingrained than first-order barriers (Ertmer, 1999). Moreover, research indicates second-order barriers are prevalent among teachers (e.g., Hermans, Tondeur, Valcke, & van Braak, 2008). In order to assist teachers to better overcome both types of barriers, we must understand the relationships that exist among them. In this paper, we share the results of our study in which we examined relationships between prospective teachers' (PTs') knowledge of how to use technology in the teaching and learning of geometry (first-order barriers) and their beliefs about the nature of mathematics, teaching, learning, and the use of technology (second-order barriers).

Related Literature Review and Theoretical Framework

Researchers (e.g., Cooney & Wilson, 1993; Pajares, 1992; Wilkins, 2008) have examined the relationships between beliefs and knowledge and found that one seems to influence the other. Pajares (1992) stated "beliefs influence knowledge acquisition and interpretation, task definition and selection, interpretation of course content, and comprehension monitoring" (p. 328). In other words, teachers' beliefs will influence the ways in which their knowledge is created and their instructional decisions. Conversely, Cooney and Wilson (1993) stated, "beliefs may be dependent on the existence or, perhaps, the absence of knowledge"(p. 150). Therefore, teachers' mathematical knowledge may lead to particular beliefs about the way that mathematical knowledge is best taught (Wilkins, 2008).

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Thus, it seems there is a bi-directional relationship between teachers' beliefs and knowledge such that they influence each other. And, both beliefs and knowledge seem to influence teachers' decisions in planning and practice.

Ernest (1989) noted that teachers' approaches to mathematics teaching related profoundly to their system of beliefs. Ernest (1989) provided a model for conceptualizing teachers' beliefs. Ernest's (1989) classifications are organized by the possible ways that teachers may view (1) the nature of mathematics, (2) teaching and (3) learning mathematics. Goos, Galbraith, Renshaw, and Geiger(2003) developed categories, Servant, Master, Partner, and Extension of Self, to describe the different ways that teachers may use technology. We use Goos et al.'s(2003) categories as ways that prospective teachers may view the use of technology for teaching and learning mathematics (see Table 1).

Table 1: Beliefs Classifications

Beliefs about	Classification of beliefs	Description
Nature of Mathematics Ernest (1989)	**Instrumentalist**	Mathematics is a set of facts and rules
	Platonist	Mathematics as a unified body of knowledge that does not change
	Problem Solving	Mathematics as a human creation that is continually changing
Teacher's Role Ernest (1989)	**Instructor**	Goal of instruction is for students to master skills
	Explainer	Goal of instruction is for students to develop conceptual understanding of a unified body of knowledge
	Facilitator	Goal of instruction is for students to become confident problem solvers
Learning Ernest (1989)	**Passive Reception of Knowledge**	Child exhibits compliant behavior and masters skills. Child passively receives knowledge from the teacher
	Active Construction of Knowledge	Child actively constructs understanding. Child autonomously explores self interests
Learning and Teaching Mathematics with Technology Goos, et al. (2003)	**Master**	Dependence on technology, not capable of evaluating the accuracy of the output generated by technology
	Servant	Fast, reliable replacement for mental or pen and paper calculations
	Partner	Cognitive reorganization, use technology to facilitate understanding, to explore different perspectives
	Extension of Self	Incorporate technological expertise as a natural part of mathematical and/or pedagogical repertoire

One framework that has become popular in recent years to describe and examine teachers' knowledge is Mishra and Koehler's (2006) Technology, Pedagogy, and Content Knowledge Framework, known as TPACK. The TPACK framework consists of three main components: Content, Pedagogical, and Technological Knowledge (CK, PK, and TK), and the intersections between and among them, represented as Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Technological, Pedagogical, Content Knowledge (TPCK).

Findings from research on the relationship between teachers' beliefs and their TPACK or integration of technology indicate that there are varying degrees of consistency between the two. Kim, Kim, Lee, Spector, and DeMeester (2013) showed that teachers' beliefs about the nature of

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content knowledge and learning and about effective ways of teaching influenced their technology integration practices. On the other hand, Chai, Chin, Koh, and Tan (2013) revealed discrepancies between participants' pedagogical beliefs and their TPACK. Chai and colleagues found teachers used a more traditional teaching practice aimed at knowledge acquisition when using technology even though many teachers held constructivist-oriented pedagogical beliefs. The use of a traditional teaching practice is likely due to teachers' lack of knowledge of how to effectively integrate technology into their classroom. Given the mixed results of research studies on the relationships that may exist between teachers' beliefs and their TPACK, additional research is needed to examine whether relationships, in fact, do exist and to describe those relationships. The purpose of this study is to investigate and describe ways in which prospective middle grades mathematics teachers' beliefs relate to their knowledge of technology, pedagogy and content. The research question that guided our study is: What are the relationships among middle grades prospective mathematics teachers' beliefs about the nature of mathematics, teaching, learning, and the use of technology in the mathematics classroom and their level of knowledge of the TPACK components in the context of geometry?

Methods

Our participants were four undergraduate PTs enrolled in a middle grades teacher education program at a university in the southeast United States. We collected data from two semi-structured interviews. In the first interview, we asked participants questions to uncover their beliefs about the nature of mathematics, teaching and learning mathematics, and the use of technology to learn and teach mathematics. In the second interview, we examined participants' TPACK by conducting a task-based interview (Hollebrands & Smith, 2010). The PTs completed four separate tasks in which they analyzed students' work in a Dynamic Geometry Environment (DGE) and created an activity using the DGE to assist students to develop a deeper understanding of the concept or to remedy students' misconceptions (see Figure 1). We analyzed the first interview using Ernest's (1989) classifications of beliefs and Goos et al.'s (2003) perspectives of technology (see Table 1). Independently, each member of the research team read the transcripts and classified each participant's beliefs according to Ernest's (1989) and Goos et al.'s (2003) categories. We shared and discussed our classifications for each belief category and came to an agreement. Next, one member of the research team wrote a brief narrative that described the participant's beliefs and justified our classification of his or her beliefs. The lead researcher performed a member check (Creswell, 2013) by sharing the narrative with each participant. Each participant agreed that we accurately captured his or her beliefs. For the second interview, we used rubrics designed by Hollebrands & Smith (2010) to score the PTs' work on each of the four tasks. The tasks and rubrics were designed to assess the PTs' levels of CK, PCK, TCK, and TPCK (see Table 2). Based on the participants' work on each of the tasks, one of four levels

TASK 1

Suppose students in your middle or high school mathematics class are studying rectangles and squares. They open a dynamic geometry sketch that contains a rectangle and a square, each of which have been constructed. Students are asked to consider properties of rectangles and squares, based on their exploration of the sketch. One pair of students has measured the diagonals and they have noticed they are always congruent. They claim, "quadrilaterals have congruent diagonals."

a. Is this claim always true, sometimes true, or never true? Explain.
b. How would you characterize their current level of geometric understanding?
c. Create a sketch using a dynamic geometry environment that you would like students to use to explore diagonals of quadrilaterals. Be sure to include directions and/or questions you would provide to students as they use this sketch.

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Figure 1.Example Task for Examining Participants' TPACK (Hollebrands & Smith, 2010)

Table 2: Rubric Used to Analyze TPACK Interview Task 1

Content Knowledge	Pedagogical Content Knowledge	Technological Content Knowledge	Technological Pedagogical Content Knowledge
A. Responds that the claim is sometimes true. B. Knowledge that there exists at least one quadrilateral for which the diagonals are not always congruent. C. States that for at least the rectangle and square the diagonals are always congruent. D. Provides a correct mathematical justification for why the statement is sometimes true using proofs that involve triangles or other properties.	A. Identifies that the student is able to notice that for a square and a rectangle that the diagonals are always congruent based on their measures. B. Identifies that the student is at level 2 (descriptive) but probably not at level 3. C. Has students consider at least one counterexample of a quadrilateral that has congruent diagonals. D. Asks students to consider at least one example of a quadrilateral that has congruent diagonals.	A. Accurately constructs or draws a quad using a DGE that is a counter-example. B. Uses measures to find the lengths of the diagonals. C. Drags to create multiple examples in a DGE. D. Accurate constructions of the 2 of the following quads: • Square • Rectangle • Parallelogram • Rhombus	A. Uses the DGE technology to focus students on properties of different quadrilaterals and their relationships to the diagonals in the task. B. Creates more than a single example using DGE technology to show the student that they are incorrect in the task. C. Designs an exploration for students by creating accurate constructions and utilizing the measurement and dragging features
Emergent: 0 or no response. Beginner: 1 of A – D Intermediate: 2 of A – D Advanced: 3 of A – D	Emergent: 0 or no response. Beginner: 1 of A – D Intermediate: 2 of A – D Advanced: 3 of A – D	Emergent: 0-1 of A – D or no response. Beginner: 2 of A – D Intermediate: 3 of A – D Advanced: All of A – D	Emergent: 0 of A – C or no response. Beginner: 1 of A – C Intermediate: 2 of A – C Advanced: All of A – C

(Beginner, Emergent, Intermediate, Advanced) was assigned for each of the four TPACK categories. We coded each participant's beliefs and TPACK levels individually, then compared codes and reconciled differences.

Findings

Content

We found that participants' level of CK aligned with their views of the nature of mathematics. Ken viewed mathematics as a set of facts, rules, and procedures that are to be utilized to solve problems. He said, "I guess personally I would define it as being able to, again using numbers in a formula to solve problem." Thus, we classified his beliefs about the nature of mathematics as Instrumentalist. In the task interview, he was able to name various shapes but could not consistently identify properties, develop counter-examples, or justify the relationships among properties. We

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coded Ken's CK as Beginner. In the beliefs interview, Kim, May, and Sue held, in part, a Problem Solving view of mathematics. May said, "I also think math can seem like really rigid and like there is one right answer and this is how you do it. So I would also see the role as a mathematics teacher is kind of dispelling some of those myths about math." Sue initially stated she viewed mathematics as a unified body of knowledge. However, her college courses have influenced her view of mathematics such that she also viewed it as a process of inquiry in which the student creates the mathematics. Thus, Sue seemed to have both Platonist and Problem Solving views. The three PTs viewed mathematics to be, in part, created by the human mind and, thus, is constantly evolving for the learner. The three PTs also displayed high levels of CK. Kim, May, and Sue were able to identify properties of different figures, relationships between figures, and determine whether a figure was rotated or reflected by examining the orientation of the figure. While May and Sue were able to justify why certain conjectures were true, Kim, at times, struggled in this area. Thus, we classified May and Sue having an Advanced level of CK and Kim having an Intermediate level of CK. Thus, the PTs who held a Problem Solving view of mathematics displayed a high level of CK, while the PT who held an Instrumentalist view displayed a low level of CK.

Pedagogy

We found two potential relationships between the PTs' beliefs about teaching and learning and their level of PCK even though their beliefs were not the same. Ken believed students learn mathematics best when the teacher is in control of distributing the content, that is, he believed students learn mathematics through a Passive Reception. During the Task interview, Ken proposed activities that could help students see their mistakes, but would not allow students to determine why they were mistaken. Rather, his activities consisted of telling them what to do. Thus, we coded Ken at a Beginner level of PCK. The three other PTs held, in part, an Active view of learning mathematics. Kim thought students should be given the opportunity to pursue their own solution paths, to solve the puzzle in their own way. May also emphasized students should have ownership of their solutions to problems. She said, "[Students] can figure it out on their own and so I feel like they'd be more likely to understand it because they made that discovery themselves, instead of me just giving it to them." Thus, Kim and May viewed learning as an Active Construction of Knowledge. Sue believed students learn mathematics best through repetition, but she learns best through discovery. Sue seemed to hold both Passive and Active views of learning depending who is doing the learning. On the task interview, Kim displayed an Intermediate level of PCK. She analyzed students' thinking and made reasonable claims about students' understanding and how they came to that understanding. However, she did not consistently create activities that would help students fully understand why a certain conjecture was true or false. Both May and Sue demonstrated advanced levels of PCK. They identified students' levels of understanding and their misconceptions. The examples, counterexamples, and exploration tasks May and Sue developed would help students recognize whether their conjectures were true and allow them to deepen their understanding of the content. Therefore, the PT who held a Passive view about learning mathematics displayed a low level of PCK, whereas the PTs who held more of an Active view of learning displayed high levels of PCK.

There also seems to be a relationship between the PTs' beliefs about teaching and their levels of PCK. Ken believed that good teachers are those who explain mathematical concepts well. He said, "I think in order to be a good math teacher you need to be able to show those students different ways of solving a problem. I think you also need to be able to show them the longer ways of why formulas work." Thus, we coded Ken's view of teaching as an Explainer. Kim and May believed students should be given the opportunity to pursue their own solution paths to become better problem solvers. Thus, they wanted to engage their future students in learning mathematics using exploratory activities. Sue said teachers should facilitate discovery, encourage problem solving, and differentiate

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learning in the classroom. Therefore, Kim, May, and Sue believed that teachers should be Facilitators. As we mentioned above, Kim's level of PCK was at the Intermediate level, and May's and Sue's were at the Advanced level, all higher than Ken's. Thus, the PT who viewed the role of a teacher as an Explainer displayed a low level of PCK, whereas the PTs who viewed a teacher as a Facilitator displayed a high level of PCK.

Technology

Comparing the participants' beliefs about the use of technology in the teaching and learning of mathematics with their levels of TCK and TPCK, two relationships emerge. First, there appears to be a relationship between teachers' beliefs about the use of technology in the mathematics classroom and their knowledge of how to use it to teach students mathematics (TPCK). Ken, Kim, and Sue's beliefs about the use of technology in the learning and teaching of mathematics aligned, in part, with the Servant role; technology is to be used to amplify cognitive process, but not change the nature of the activities (Goos et al., 2003). In the task interview, the three PTs struggled to develop technology-based activities to help students overcome a misconception and the activities they created were limited to a certain number of examples or they did not use some of the basic features of the tool. We classified their TPCK at the Beginner level. May viewed technology as a Partner; technology should be used to explore and deepen students understanding of mathematics. In the task interview, May's activities focused on correcting students' misconceptions and having students understand and consider fundamental properties and relationships that would deepen their understanding of mathematics. We coded May's level of TPCK as Advanced. Thus, it appears that the PT who viewed technology as a Partner displayed a high level of TPCK while the PTs who viewed technology as a Servant displayed low levels of TPCK.

Second, there does not seem to be a relationship between their beliefs about the use of technology in the teaching and learning of mathematics and their level of TCK. Ken and Kim held very similar views that technology could be used as both a Servant and a Partner. We coded both Ken and Kim at a Beginner level of TCK because they could use multiple tools and features of the DGE (e.g., perform constructions of figures, measure different components of the figures, use the drag feature, and label points), but they did not use them consistently across all four tasks and struggled to accurately construct some figures. May viewed technology as a Partner. During the Task interview, May displayed an Intermediate level of TCK, but she may hold a more advanced level of knowledge because she was unable to use the technology on the final task due to time constraints. Sue's beliefs about the use of technology aligned with the Servant view. During the task interview, Sue used multiple features of the DGE for the majority of the tasks but struggled performing transformations. Thus, Sue had an intermediate level of TCK. Ken, Kim, and May's views of technology seemed to correspond with their level of knowledge of how to use technology; the greater the PT's TCK, the stronger the belief in using it as a way to engage students in learning mathematics. However, Sue's view of technology and her knowledge of how to use it did not follow this relationship. Thus, we cannot state with any degree of certainty that a relationship exists between teachers' TCK and their beliefs about technology.

Discussion, Limitations, & Implications

The goal of this study was to investigate the relationships among prospective middle grades mathematics teachers' TPACK in the context of geometry (first-order barriers) and their beliefs about the nature of mathematics, learning and teaching mathematics, and the use of technology (second-order barriers). We found that PTs' beliefs about the nature of mathematics relates to their CK, their beliefs about learning and teaching mathematics relates to their PCK, and the use of technology to teach mathematics is related to their TPCK. May was the only PT who displayed high levels in all of

the TPACK components. Sue displayed the same levels as May for CK, PCK, and TCK. However, she displayed a low level of TPCK. May and Sue's beliefs were not the same either. May displayed student-centered views of learning and teaching while Sue seemed to have both teacher-centered and student-centered views. In addition, Sue had a servant view of technology. Sue's low level of TPCK is likely related to her beliefs. Sue struggled in developing appropriate technology based activities for students because she had not developed appropriate views of effectively teaching with technology. Kim et al. (2013) found teachers' integration practices differed even though the teachers had access to the same technologies, support, and training. Teachers who held more student-centered views were able to integrate technology more seamlessly into their practices than those with more teacher-centered views. Even though the teachers had a similar knowledge base, the researchers found their integration practices differed and their beliefs seemed to influence the integration. Based on the findings from both our study and Kim et al. (2013), developing a strong knowledge of content, pedagogy, and technology will not ensure that teachers will use technology effectively in the classroom.

Unlike May, Ken's views about the nature of mathematics, learning, and teaching mathematics were teacher-centered and he displayed low levels of TCK and TPCK. This leads us to believe that in order for teachers to use technology effectively, they must develop student-centered views of learning and teaching. However, Kim has student-centered beliefs about teaching and learning, yet she displayed low levels of TPACK. This finding is consistent with Chai et al. (2013) who indicated that while their prospective teachers had developed student-centered perspectives of learning and recognized the advantages of using technology, they struggled designing appropriate technology-based activities. Thus, having student-centered beliefs will not ensure teachers will be able to use technology effectively.

There are two limitations to our study. The first limitation is the number of participants. Perhaps if we had interviewed additional or different participants, we may have found different relationships. The second limitation relates to the underpinnings of the rubric used to determine the level of TPACK knowledge displayed by our participants. Hollebrands & Smith, (2010) developed the rubric based, in part, on what research considers the best practices to teach mathematics with technology, in particular a DGE (e.g. Laborde, Kynigos, Hollebrands, & Staesser, 2006). These practices are based on constructivist principles. Thus, the rubric is likely biased towards participants with student-centered beliefs such that participants who held these beliefs would perform at high level compared to those who held teacher-centered beliefs. In fact, Ken, who held teacher-centered beliefs, did not achieve high levels while May, who holds student-centered beliefs, displayed high levels of TPACK. However, Kim and Sue held, in part, student-centered beliefs and did not display high levels for all TPACK components. Thus, the relationships between the participants' beliefs and TPACK were not predetermined. Even though there are limitations to this study, we believe there are implications for researchers and educators.

As technology becomes more ubiquitous in the classroom, first-order barriers will persist; software developers and curriculum designers are constantly adding new features and creating new tools for the classroom each year. Teachers will need training and support to integrate these updated and new tools into their classroom. However, our findings indicate that just providing knowledge on how to use these tools will not be enough. In order for teachers to use technology effectively, mathematics teacher educators should also focus on developing teachers' beliefs.

In this study, we examined the relationships that may exist between prospective middle grades teachers' beliefs and their TPACK, although we only analyzed their work with a particular tool, a DGE. Future work should examine whether the relationships we found would be the same for other technological tools. In addition, we wondered if the same relationships would appear for prospective

secondary mathematics teachers and practicing teachers. We believe further research is needed in this area to provide a clearer picture of the relationships between teachers' beliefs and their TPACK.

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